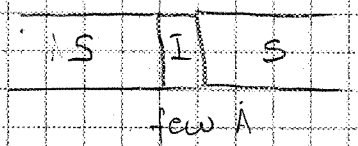
Josephson Effect



Al $\quad Al_2O_3 \quad$ Al

tunnel effect:

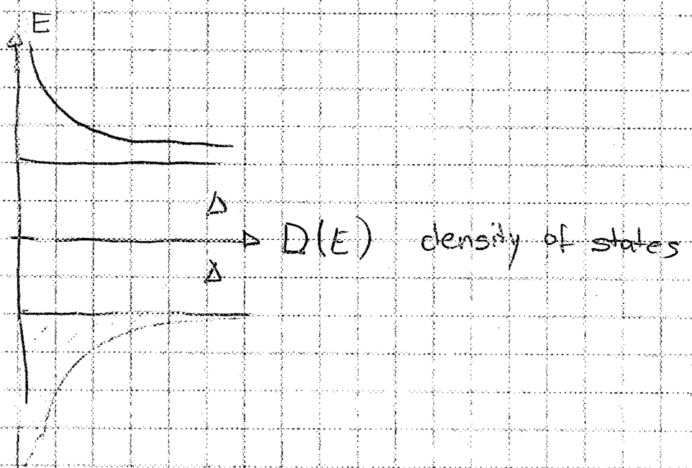
$e^{-\kappa x} \qquad \kappa = \frac{1}{\hbar}\sqrt{2m(V_0 - E)}$

superconductivity

Bardeen Cooper Schrieffer

Cooper pairs $2e \qquad (\vec{k}, -\vec{k})$

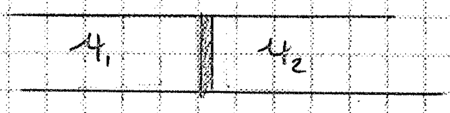
condensate $\quad \psi = \sqrt{n}\, e^{i\theta}$



superconductivity class

Zehetmayer / Weber

DC Josephson Effect



$$i\hbar \frac{\partial}{\partial t}\psi_1 = \hbar T \psi_2 \qquad i\hbar \frac{\partial}{\partial t}\psi_2 = \hbar T \psi_1$$

$$\psi_1 = \sqrt{n_1}\, e^{i\theta_1} \qquad \psi_2 = \sqrt{n_2}\, e^{i\theta_2} \qquad n_1, \theta_1 \in \mathbb{R}$$

$$\frac{\partial}{\partial t}\psi_1 = \frac{1}{2} n_1^{-1/2} e^{i\theta_1} \frac{\partial n_1}{\partial t} + i\, n_1^{1/2} e^{i\theta_1} \frac{\partial \theta_1}{\partial t} = -iT\psi_2$$

$$\frac{\partial}{\partial t}\psi_2 = \frac{1}{2} n_2^{-1/2} e^{i\theta_2} \frac{\partial n_2}{\partial t} + i\, n_2^{1/2} e^{i\theta_2} \frac{\partial \theta_2}{\partial t} = -iT\psi_1$$

multiply by $n_1^{1/2} e^{-i\theta_1}$

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = -iT (n_1 n_2)^{1/2} e^{i\delta} \qquad \delta = \theta_2 - \theta_1$$

$$\frac{1}{2}\frac{\partial n_2}{\partial t} + i n_2 \frac{\partial \theta_2}{\partial t} = -iT (n_1 n_2)^{1/2} e^{-i\delta}$$

real part — imaginary part

$$\frac{\partial n_1}{\partial t} = 2T (n_1 n_2)^{1/2} \sin\delta \qquad \frac{\partial \theta_1}{\partial t} = -T\left(\frac{n_2}{n_1}\right)^{1/2} \cos\delta$$

$$\frac{\partial n_2}{\partial t} = -2T (n_1 n_2)^{1/2} \sin\delta \qquad \frac{\partial \theta_2}{\partial t} = -T\left(\frac{n_1}{n_2}\right)^{1/2} \cos\delta$$

equal superconductors $n_1 \cong n_2$

$$\frac{\partial \theta_1}{\partial t} = \frac{\partial \theta_2}{\partial t} \qquad \frac{\partial}{\partial t}\delta = 0$$

$$\frac{\partial n_2}{\partial t} = -\frac{\partial n_1}{\partial t}$$

L6.1

$$I = 2e\frac{\partial n_1}{\partial t}$$

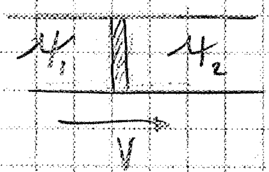
$$I = I_0 \sin(\delta) = I_0 \sin(\theta_2 - \theta_1)$$

DC Josephson effect

DC supercurrent without voltage

I_0 junction property
$\propto$ exp. thickness
$\propto$ A

AC Josephson effect



$$i\hbar \frac{\partial}{\partial t}\psi_1 = \hbar T \psi_2 - eV\psi_1$$
$$i\hbar \frac{\partial}{\partial t}\psi_2 = \hbar T \psi_1 + eV\psi_2$$

$$\frac{1}{2}\frac{\partial n_1}{\partial t} + i n_1 \frac{\partial \theta_1}{\partial t} = \frac{i}{\hbar} eVn_1 - iT(n_1 n_2)^{1/2} e^{i\delta}$$

$$\frac{\partial n_1}{\partial t} = 2T(n_1 n_2)^{1/2}\sin(\delta)$$

$$\frac{\partial \theta_1}{\partial t} = \frac{eV}{\hbar} - T(n_1 n_2)^{1/2}\cos\delta$$

$$\boxed{\frac{\partial}{\partial t}(\theta_2 - \theta_1) = \frac{\partial}{\partial t}\delta = 2eV/\hbar}$$

AC Josephson effect

$$I = I_0 \sin\left(\delta(0) - \frac{2eV}{\hbar} t\right)$$

cooper pairs

$$\omega = \frac{2eV}{\hbar} \qquad \hbar\omega = 2eV$$

Voltage standard

$$\nu = \frac{2 \cdot 1{,}6 \cdot 10^{-19} \cdot 10^{-6}}{6{,}6 \cdot 10^{-34}} \approx \frac{1}{2} \cdot 10^{9} \approx 500\ \text{MHz}$$

$$V = \frac{\phi_0}{2\pi} \dot{\delta} \qquad \phi_0 = \frac{h}{2e} \quad \text{superconducting flux quantum}$$

$$= 2 \cdot 10^{-15}\ \text{Wb} = 20\ \text{Gauss}\ (\mu m)^2$$

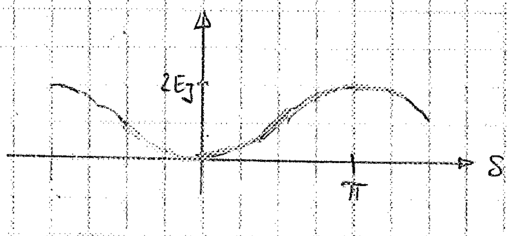
$$\text{Wb} = \text{Tesla}\ m^2$$

Josephson energy

$$E(\delta) = \int_0^t I V \, d\tau = \frac{I_0 \phi_0}{2\pi} \int_0^t \sin(\chi) \frac{d\chi}{d\tau} \, d\tau$$

$$= \frac{I_0 \phi_0}{2\pi} \int_0^{\delta} \sin(\chi) \, d\chi = \frac{I_0 \phi_0}{2\pi} \left(1 - \cos(\delta)\right)$$

$$\boxed{E = E_J \left(1 - \cos(\delta)\right) \qquad E_J = \frac{I_0 \phi_0}{2\pi}}$$



Josephson inductance

$$L = \frac{V}{\dot{I}} = \frac{\phi_0/2\pi \ \dot{\delta}}{I_0 \frac{d}{dt}\left(\sin(\delta)\right)} = \frac{\phi_0}{2\pi I_0} \frac{\frac{d\delta}{dt}}{\cos(\delta) \frac{d\delta}{dt}}$$

$$\boxed{L = \frac{\phi_0}{2\pi I_0} \frac{1}{\cos(\delta)}}$$

Lb.3

gauge change & phase

Hamiltonian for a charged particle in a EM field

$$H = \frac{(\vec{p} - q\vec{A})^2}{2m} + q\Phi$$

physical fields

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

change gauge: scalar field $\chi(\vec{x}, t)$

unit $[\chi]$ = Volt·sec

$$\vec{A} \longrightarrow \vec{A}' = \vec{A} + \vec{\nabla}\chi$$

$$\phi \longrightarrow \phi' = \phi - \frac{\partial \chi}{\partial t} \qquad \psi \longrightarrow \psi' = \psi \cdot e^{i\chi q/\hbar}$$

gauge invariant phase difference

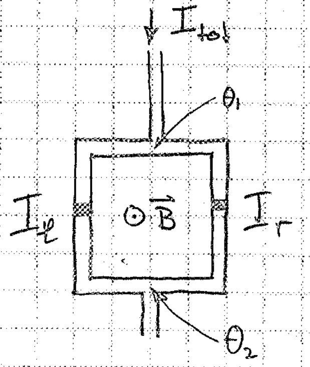
$$\delta = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0}\int_1^2 \vec{A}\, d\vec{\ell}$$

$$\delta \longrightarrow \delta' = \theta_2 + \chi(\vec{x}_2, t)\frac{2e}{\hbar} - \theta_1 - \chi(x_1, t)\frac{2e}{\hbar}$$

$$- \frac{2\pi}{\phi_0}\int_1^2 (\vec{A} + \vec{\nabla}\chi)\, d\vec{\ell} \quad = \delta$$

using $\quad \frac{2\pi}{\phi_0}\int_1^2 \vec{\nabla}\chi\, d\vec{\ell} = \frac{2\pi}{\phi_0}\chi\Big|_1^2 = \frac{2\pi}{\phi_0}\left(\chi(\vec{x}_2) - \chi(\vec{x}_1)\right)$

SQUID - superconducting quantum interference device



$$I_{total} = I_\ell + I_r = I_0 \sin(\delta_\ell + I_0 \sin(\delta_r)$$

assumed equal junctions

$$\delta_\ell = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0} \underbrace{\int_1^2 \vec{A}\, d\vec{\ell}}_{\text{left path}}$$

$$\delta_r = \theta_2 - \theta_1 - \frac{2\pi}{\phi_0} \underbrace{\int_1^2 \vec{A}\, d\vec{\ell}}_{\text{right path}}$$

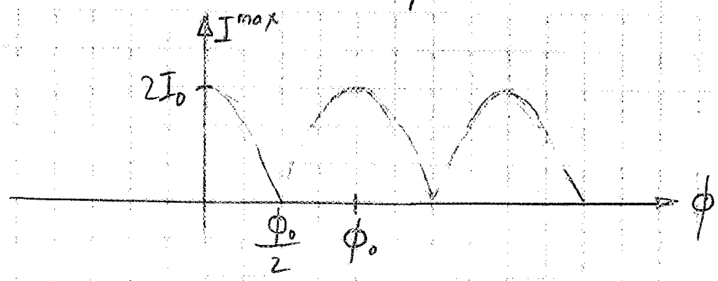
$$I_{total} = 2 I_0 \sin\left(\frac{\delta_\ell + \delta_r}{2}\right) \cos\left(\frac{\delta_\ell + \delta_r}{2}\right)$$

$$= 2 I_0 \sin\left(\frac{\delta_\ell + \delta_r}{2}\right) \cos\left(\frac{\pi}{\phi_0} \oint \vec{A}\, d\vec{\ell}\right)$$

using $\oint \vec{A}\, d\vec{\ell} = \int_S \vec{\nabla} \times \vec{A}\, d\vec{\sigma} = \int_S \vec{B}\, d\vec{\sigma} = \phi$

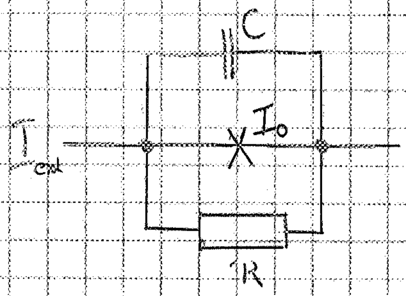
maximal supercurrent, i.e. switching current

$$I_{total}^{max} = 2 I_0 \left|\cos\left(\pi \frac{\phi}{\phi_0}\right)\right|$$



L6.5

RCSJ Model (resistively and capacitively shunted junction)



$$I = I_0 \sin(\delta) + \frac{V}{R} + C\frac{dV}{dt} \qquad \left(\begin{array}{c} Q = CV \\ I = \dot{Q} \end{array} \right)$$

$$V = \frac{\phi_0}{2\pi}\dot{\delta}$$

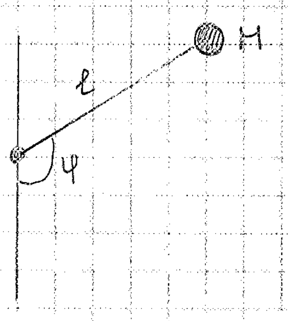
$$I = I_0 \sin(\delta) + \frac{\phi_0}{2\pi R}\dot{\delta} + \frac{C\phi_0}{2\pi}\ddot{\delta}$$

$$\tau = \omega_p t \qquad \omega_p = \sqrt{\frac{I_0 2\pi}{C\phi_0}} = \sqrt{\frac{I_0 2e}{\hbar C}}$$ plasma frequency

$$Q = \omega_p RC \qquad \beta_c = Q^2$$ McCumber parameter

$$\frac{d^2\delta}{d\tau^2} + \frac{1}{Q}\frac{d\delta}{d\tau} + \sin(\delta) = \frac{I}{I_0} \qquad Q \propto \sqrt{I_0 C}\, R$$

pendulum with friction



homework:
- derive equation of motion
- identify terms

tilted washboard model

particle of mass $m_p = \left(\frac{\phi_0}{2\pi}\right) C$ with

viscous drag force $f_{fr} = \left(\frac{\phi_0}{2\pi}\right)\frac{1}{R}$

moving in an effective potential

$$U(\delta) = -E_J \cos(\delta) - \frac{\phi_0}{2\pi} I \delta$$

slide
"tilted washboard potential"

$$m_p \ddot{\delta} + f_{fr}\dot{\delta} - \frac{\partial U}{\partial \delta} = 0$$

for $I < I_0$ static solution

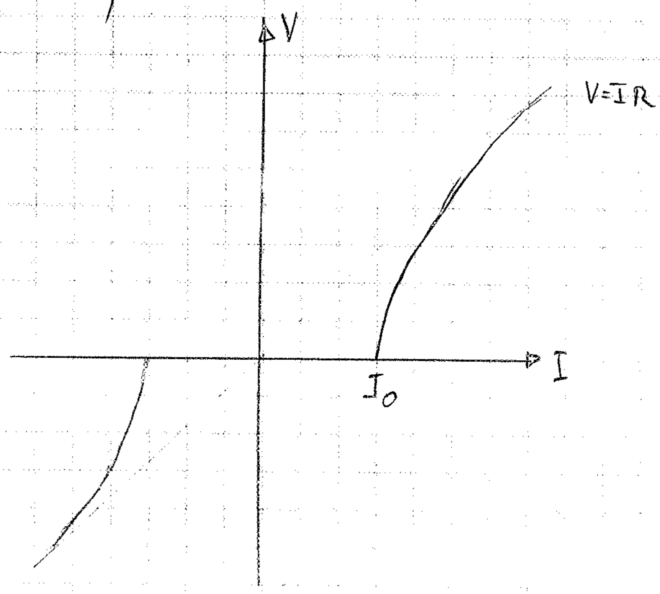
$$\delta = \arcsin\left(\frac{I}{I_0}\right) \qquad V = 0$$

overdamped $Q \ll 1$

$$\dot{\delta} = \frac{2\pi I_0 R}{\phi_0}\left(\frac{I}{I_0} - \sin\delta\right)$$

average voltage:

$$V = R\sqrt{I^2 - I_0^2}$$



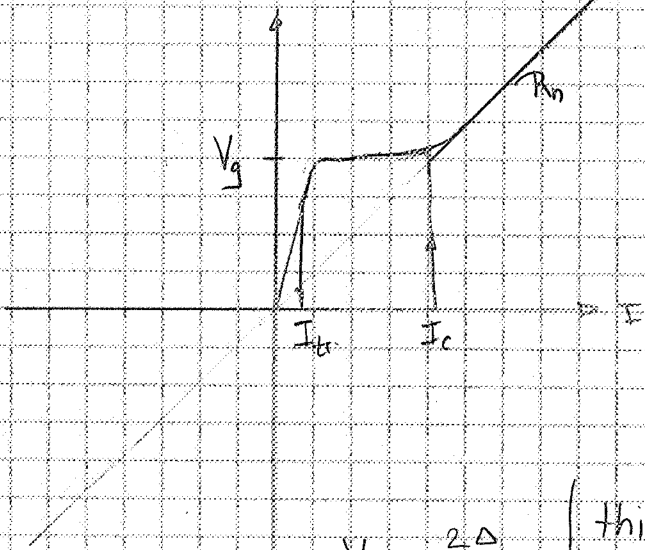
which can be found by integration over one periode

$$\int_0^{2\pi} \frac{d\delta}{\frac{I}{I_0} - \sin\delta} = \frac{2\pi I_0}{\phi_0} R \int_0^T dt$$

$$\frac{2\pi}{\sqrt{\left(\frac{I}{I_0}\right)^2 - 1}} = 2\pi \frac{I_0}{\phi_0} R T \qquad V = \frac{\phi_0}{2\pi}\frac{1}{T} = R\sqrt{I^2 - I_0^2}$$

underdamped $Q > 1$

hysteretic behaviour

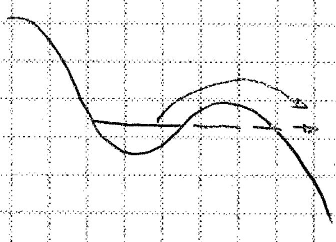


$$V_g = \frac{2\Delta}{e}$$ (this is not described with a constant R)

$$I_{tr} = 4\, I_0 \frac{1}{\pi Q}$$

$$I_c R_n = \pi \frac{\Delta}{2e}$$

thermal fluctuations
quantum fluctuations
} { reduce switching current
increase retrapping current



thermal excitation over the barrier

quantum tunneling through the barrier